

A Higher-Order Calculation of np Scattering in Cut-Off Effective Field Theory

Chang Ho Hyun^{a,1}, Tae-Sun Park^{b,2} and Dong-Pil Min^{a,c,3}

^a*Department of Physics, Seoul National University, Seoul 151-742, Korea*

^b*Theory Group, TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3*

^c*Service de Physique Théorique, CE Saclay, 91191 Gif-sur-Yvette, France*

Abstract

We report a next-to-leading-order (NLO) chiral perturbation theory calculation of the neutron-proton scattering cross section in the 1S_0 channel using a cut-off regularization. The inclusion of two-pion exchanges in the irreducible diagrams – or potential – figuring at NLO is found to be important in enlarging the domain of validity of the effective field theory. We are able to reproduce the scattering phase shift data up to $p = 300$ MeV, with an agreement which is superior to results of other effective field theory approaches. We also discuss the importance of the explicit pion degree of freedom in scattering process.

PACS: 03.65.Nk; 13.75.Cs; 11.15.Bt.

Keywords: Effective field theory; Neutron-proton scattering; Finite cutoff scheme.

¹e-mail: hch@zoo.snu.ac.kr

²e-mail: park@alph02.triumf.ca

³e-mail: dpmin@phya.snu.ac.kr

In a series of recent publications, two of the authors (DPM and TSP) together with Kubodera and Rho [1, 2, 3] presented quite successful examples of effective field theory in nuclear physics, by showing response functions to electroweak processes at low energy involving two-nucleon systems can be described quite accurately with little cutoff dependence. (For others' works on this area, see Refs. [4, 5, 6, 7, 8].) The approach used there was the “ Λ counting scheme” proposed by Weinberg [9]. The focus on the response functions instead of on scattering observables is in the spirit of the long-standing tradition in nuclear physics where the wealth of nuclear dynamics has been accumulated more powerfully through response functions than through scattering processes per se.

The recent surge of activity in effective field theories in nuclear physics [10] was however triggered by the effort to give a first-principle description of the large scattering lengths in nucleon-nucleon scattering, so a large portion of the recent publications has been devoted to scattering amplitudes at low energy. “ Q counting” scheme introduced by Kaplan, Savage and Wise [11] using power divergence subtraction (PDS) exemplifies the present preoccupation of the workers in this field.^{#1} While there appeared a few papers on the power of the Λ scheme in scattering [13, 14, 15], an exhaustive confrontation of the Λ scheme developed for the response functions [1] with scattering amplitudes has not been performed. It is the purpose of this paper to provide the missing information. We shall show that the scheme is as successful for scattering as for response functions and in fact is even more accurate than the Q counting scheme.

The question we address is this: How does the Λ scheme of [1] which is remarkably successful in postdicting and predicting electroweak processes of two-nucleon systems $p+p \rightarrow d + e^+ + \nu$ [2] and $n + p \rightarrow d + \gamma$ [3, 16] fare with the two-nucleon scattering and to what nucleon momentum and with what accuracy can one “push” the scheme?

For np scattering to the next-to-leading order (NLO) in the Λ scheme with pions and nucleons, the two-particle irreducible graphs that generate the potential for the system with which the Schrödinger(or Lippman-Schwinger) equation is to be solved comprise of nucleonic contact interactions and pion-exchange potentials. The contact terms, the coefficients of which will be fit to empirical data at the zero momentum limit, represent the short-range part of the nuclear interaction. On the other hand, the pion exchange potentials control the long range part of the nuclear interaction, so unsurprisingly the explicit inclusion of the pion-exchange potential generally increases the range over which the wave functions are accurate. It was shown in [1] that the absence of the OPEP could barely be compensated by the leading-order contact term and that the contact interaction at the next-to-leading order (NLO) – when added to the LO interaction – is seen to improve considerably the stability of the numerical results, thereby increasing the domain of validity of the effective field theory. In this paper, we will take explicitly into consideration the two-pion-exchange potential(TPEP) at the NLO and show how accurately one can calculate the two-nucleon

^{#1}There is also a slightly different scheme which emphasizes the unitarity and the relativistic formalism suggested by Lutz [12].

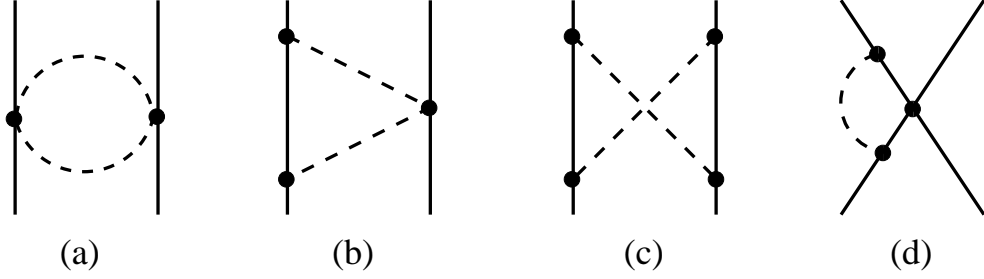


Figure 1: Two-body irreducible diagrams that contribute to the TPE potential.

process. In addition, we will also report on a higher order calculation.

We have described in great detail the main strategy of the cut-off EFT in our previous works, so we will not enter into it here. It will suffice to briefly define the convention and go directly into the results. We adopt the power counting rule given in [9]: an irreducible diagram is of order of $\mathcal{O}((Q/\Lambda_\chi)^\nu)$ where Q is the pion mass or the typical momentum scale of the process and $\Lambda_\chi \sim 1$ GeV. And we keep only pions and nucleons as pertinent degrees of freedom, all other massive degrees of freedom are integrated out. In nucleon-nucleon potential, the LO has $\nu = 0$ and consists of OPEP plus non-derivative contact interactions:

$$\mathcal{V}_{LO}(\vec{q}) = \frac{4\pi}{M} C_0 + \mathcal{V}_{1\pi}(\vec{q}), \quad \mathcal{V}_{1\pi}(\vec{q}) = \frac{g_A^2}{4f_\pi^2} \frac{\vec{q}^2}{m_\pi^2 + \vec{q}^2}, \quad (1)$$

where M is the nucleon mass, $g_A \simeq 1.25$ the axial-vector coupling, $f_\pi \simeq 93$ MeV the pion decay constant and \vec{q} the momentum transfer. There is no $\nu = 1$ contribution, the NLO ($\nu = 2$) comes from TPEP and contact interactions with two derivatives. TPEP contains also $\nu = 3$ (and higher order) contributions, which correspond to the subleading order $\pi\pi NN$ vertices. There have been some elaborate works on the TPEP [17, 18, 19] up to $\nu = 3$ order and applications of the potential in other channels as well as the S -wave np scattering. In this work, we will not consider the $\nu = 3$ order contributions, though it may results in substantial improvement of the theoretical predictions.

The *two-nucleon irreducible* diagrams that contribute to the TPEP are shown in Fig. 1. In addition, we also include the contributions from OPEP-subtracted two-pion-box diagrams, whose effects are negligible in the 1S_0 channel. Remembering that the contact interaction with the coefficient C_2 enters at the same order, we write the full NLO potential as

$$\mathcal{V}_{NLO}(\vec{q}) = \frac{4\pi}{M} C_2 \vec{q}^2 + \mathcal{V}_{2\pi}(\vec{q}), \quad (2)$$

with

$$\mathcal{V}_{2\pi}(\vec{q}) = \frac{1}{64\pi^2 f_\pi^4} \int_0^1 dx \frac{(\vec{q}^2)^2}{\frac{4m_\pi^2}{1-x^2} + \vec{q}^2} f_{2\pi}(x) \quad (3)$$

$$f_{2\pi}(x) = \frac{4x^4 + 8g_A^2 x^2(3 + 2x^2) - 4g_A^4(3 + 48x^2 + 8x^4)}{24(1 - x^2)}. \quad (4)$$

To the same order in the chiral counting, there are loop corrections in the propagators and the πNN vertex. However in the kinematics involved in the elastic scattering, they do not bring in any momentum-dependent corrections: They modify only the constants (masses and coupling constants) which are to be absorbed into the parameters extracted from experiments. ^{#2}

So far, we have assumed that $|\vec{q}|$ is of the same size of the pion mass m_π . When $|\vec{q}|$ is much smaller than m_π , we can integrate out even the pions. Theory then contains only nucleons explicitly, and the expansion parameter becomes $|\vec{q}|/m_\pi$, rather than Q/Λ_χ . As we see, OPEP and TPEP become of order of $|\vec{q}|^2/m_\pi^2$ and $|\vec{q}|^4/(2m_\pi)^4$, so they can be replaced by suitable series of contact interactions starting from two-derivatives and four-derivatives. By the same token, multi-pion-exchange potentials can also be replaced by contact interactions. Recalling that multi-pion-exchange potentials are hard to calculate, this procedure provides us an efficient way of going to higher order. The price of this simplification is the shrinking-down of the radius of convergence. Following to the general argument of the effective-range expansion, the radius of convergence is about a half of the lightest mass of the degree of freedom that is not explicitly taken. That is, without the pion degrees of freedom, we can describe only up to $p \lesssim m_\pi/2$, where p is the CM momentum. Taking OPEP explicitly with higher order contact interactions, we then expect that we can achieve accurate description of the scattering data up to $p \lesssim m_\pi$, which was indeed proved in [1]. In this paper we go to one further step, by exploring the consequences of the inclusion of the TPEP. With the TPEP, we expect we can go up to $p \lesssim \frac{3}{2}m_\pi$. Basically, in this work, we will focus on the comparison between the NLO potentials with and without the TPEP. For clarity, we shall refer to the latter as the $1\pi C_2$, meaning that it contains OPEP and contact interactions up to C_2 term. Thus in this notation, LO is identical to $1\pi C_0$ and NLO is identical to $2\pi C_2$. One of our principal findings from this comparison between the NLO and $1\pi C_2$ potentials is that the explicit presence of the pion within the Λ scheme makes the EFT a lot more versatile and accurate. In fact, this work provides a confirmation of the partial finding in [1] of the effect of the explicit OPEP in NLO and $1\pi C_2$ in scattering consideration. We will also consider $2\pi C_4$ potential, which consists of the NLO potential and four-derivative contact interaction of the form $\frac{4\pi}{M}C_4|\vec{q}|^4$. This $2\pi C_4$ potential as a possible next subleading order may only find its justification for $|\vec{q}|$ smaller than m_π , whereas for other kinematical zone, the inclusion of all subleading NNLO TPEP should be made [21]. Thus compared to the NLO, we expect that the theory can improve (both in accuracy and in cutoff-independence) results at that low momentum region, but not the radius of convergence in general. These hypotheses will be proved in our work. For the low momentum region with $p \lesssim \frac{3}{2}m_\pi$, we expect this potential provides us a simple but sufficiently adequate way to do an $\mathcal{O}(|\vec{q}|^4)$ -order calculation without including pion two-loops. It is worth noting that the integral in Eq.(3) can be rewritten in a spectral

^{#2} There are also certain contributions that come from relativistic corrections. A detailed analysis of these contributions is recently given by Friar [20]. Here we are not concerned with them.

representation by changing the integration variable x to a “spectral mass” $m \equiv \frac{2m_\pi}{\sqrt{1-x^2}}$, which runs over $2m_\pi \leq m < \infty$. One may then introduce an upper limit to the spectral mass $m \leq m_0$, by imposing the range of the integration variable to be $0 \leq x \leq \sqrt{1 - \frac{4m_\pi^2}{m_0^2}}$. The m_0 should not be confused with the genuine cutoff of the theory Λ that enters through the Fourier transformation as defined below. Note that the TPEP is negative while OPEP is positive for the whole range of $|\vec{q}|$ and that it is quite small compared to the OPEP, less than 3 % for the momentum range $|\vec{q}| \leq m_\pi$. This implies that the results obtained in [1] will not be changed appreciably by the explicit inclusion of the TPEP. As the momentum increases, however, the genuine TPEP will play an important role for the radius of the convergence, the cutoff-dependence and the allowed range of the cutoff of the theory. Throughout this work, we put the value of m_0 large enough, $m_0 = 2 \text{ GeV}$ – instead of infinity – to economize the calculation.

These potentials of various chiral orders, transformed into coordinate space, will be put into the Schrödinger equation. In doing the Fourier transform, a cutoff in momentum space will be introduced. The potential with the cutoff Λ takes the form

$$V(\vec{r}) \equiv \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} S_\Lambda(\vec{q}^2) \mathcal{V}(\vec{q}) \quad (5)$$

with

$$S_\Lambda(\vec{q}^2) = e^{-\frac{\vec{q}^2}{2\Lambda^2}}. \quad (6)$$

Due to the cutoff – which is kept finite, the integrals are finite so that we don’t need counter terms in a proper sense in addition to the contact terms that figure to the order considered. However, the concept of renormalization is still valid and figures in the requirement that the results be stable against the variation of the cutoff. The Fourier transformation of the OPEP and the contact terms can be done straightforwardly (see, for example, [1, 22]). For the TPEP, a similar technique can be used by performing the Fourier transformation before the parametric space integration. The wave function $\psi_0(r) \equiv \frac{u_0(r)}{r}$ is then obtained through the Schrödinger equation

$$\left[\frac{d^2}{dr^2} + M(E - V(r)) \right] u_0(r) = 0 \quad (7)$$

where $E = p^2/M$ is the total energy of the system, and $p = |\vec{p}|$ is the center-of-mass momentum. The solutions are to fit the *empirical* effective range expansion,

$$p \cot \delta = -\frac{1}{a_0} + \frac{1}{2}r_e p^2 + v_2 p^4 + \mathcal{O}(p^6). \quad (8)$$

For this we shall take the Nijmegen partial wave analysis (NPWA) [23] as the empirical data and extract from it the values of the low-energy constants, $a_0 = -27.73 \text{ fm}$ and $r_e = 2.677 \text{ fm}$. Note that the NPWA results are in very good agreement with the values obtained by the Argonne v_{18} potential[24], i.e., $a_0(v_{18}) = -27.732 \text{ fm}$ and $r_e(v_{18}) = 2.697 \text{ fm}$. The

$\Lambda(\text{MeV})$		200	300	400	500	600
LO	C_0	-1.27	-0.83	-0.62	-0.50	-0.42
$1\pi C_2$	C_0	-1.32	-1.21	-0.35	9.17	98.13
	$C_2\Lambda^2$	0.06	0.70	1.77	5.54	23.52
NLO	C_0	-1.31	-1.24	-1.08	0.14	11.43
	$C_2\Lambda^2$	0.11	0.85	1.74	3.27	7.48
$2\pi C_4$	C_0	-1.32	-0.68	16.40		
	$C_2\Lambda^2$	-0.12	0.75	7.12		
	$C_4\Lambda^4$	0.14	0.38	1.56		

Table 1: Coefficients of the contact interaction terms in LO, $1\pi C_2$, NLO and $2\pi C_4$ potentials, for different values of the cutoff Λ . We have multiplied Λ^n to the C_n ($n = 0, 2, 4$) so that the unit of the $C_n\Lambda^n$ is fm for any n . The blanks for the $2\pi C_4$ are explained in the text.

constant C_4 will be determined to reproduce the effective volume $v_2 = -0.48 \text{ fm}^3$ [25], which was extracted from the NPWA data [23].^{#3} We are now in position to discuss our results.

In confronting the experiment with our theory, we shall particularly be interested in assessing how well the basic tenets of an effective field theory of the type we are considering are satisfied [22]. The first is that the effective field theory in question is a nonrenormalizable theory and a truncated version with a given cutoff scale must break down at some point signaling the emergence of “new physics.” The second point is that the effective theory with a cutoff regularization is valid only if the physical observables calculated within the scheme *become stable* against the variation of the cutoff *when higher order terms are included*, which is nothing but an approximate manifestation of the renormalization group (RG) invariance that any valid effective theory must satisfy.

Let us see how these points emerge in the calculation. In Table 1 are displayed the values of the coefficients for various potentials. We can observe in the Table that, except for the LO case, the values of C ’s increase rapidly at a certain value of the cutoff. In fact there is a critical value of the cutoff beyond which we cannot find the C ’s that reproduces the low-energy constants [26]. The $2\pi C_4$ for $\Lambda \geq 500 \text{ MeV}$ is found to be such a case.^{#4} We show in Fig. 2 the 1S_0 phase shift as a function of the cutoff Λ for the initial CM momenta of 70, 140, 210 and 280 MeV (from top to bottom) for different potentials. In the figure, the NPWA data are represented by thin horizontal solid lines; the LO, $1\pi C_2$, NLO and

^{#3}In [15], a slightly different renormalization procedure has been introduced. They are using the modified effective range expansion, which deals with the OPEP-subtracted NN potential. Their method is not adopted in our work.

^{#4} If we allow a slight deviation by about 0.03 fm^3 in the effective range, we can go at least up to $\Lambda = 600 \text{ MeV}$.

$2\pi C_4$ results by dotted, dashed, solid and dotted-dashed curves, respectively. The extensive comparison between LO (dotted curves) and $1\pi C_2$ (dashed curves) in this framework has been given in [1]: By adding C_2 term into the LO potential, the $1\pi C_2$ theory becomes much more accurate with much less cutoff dependence compared to the LO theory. Indeed, even at $p = 140$ MeV where the theory is expected to break down, the theory differs from experiment only by less than 3 % for all the region with $\Lambda \geq 300$ MeV. We also observe that the theory becomes closest to the experiment at $\Lambda \sim 300$ MeV, while the LO favors $\Lambda \sim 200$ MeV. Now by adding the TPEP, we see that the NLO (solid curves) becomes more accurate than the $1\pi C_2$ at higher cutoff. For all the region considered with $\Lambda \geq 300$ MeV, the NLO differs from experiment less than 13 % at $p = 210$ MeV. Recalling that the theory is expected to break down for $p \gtrsim \frac{3}{2}m_\pi$, the above result is encouraging. It is also worthwhile observing that the NLO with $\Lambda = 400$ MeV nearly coincides with the NPWA for all the momentum considered, $p \leq 280$ MeV. On the other hand the improvement between the $1\pi C_2$ and the NLO is not spectacular. The fact that the $2\pi C_4$ cannot come closer to the NPWA shows the importance of the omitted subleading TPEP and higher order terms. Thus mere inclusion of higher-order contact interactions cannot possibly ameliorate the convergence. This indicates that any attempt to improve the precision by merely including higher order contact terms without the explicit account of the relevant degrees of freedom that are nearby is doomed to fail.

In the figure, it is clearly seen that the theories with higher terms become more insensitive to the cutoff, which shows the realization of the above mentioned RG invariance. At $p = 280$ MeV, while the NLO shows reasonable agreement with experiment, results show large fluctuations with respect to cutoff which implicates the breakdown of the theory. From Fig. 3, one can confirm that the theory breaks down at around this momentum.

For completeness, we shall now compare our cutoff scheme with the PDS scheme[27, 28]. For ease of comparison, we estimate the deviation from the NPWA result for the effective range function $\cot \delta$ of the cutoff EFT and PDS schemes as a function of the momentum. The results are shown in Fig. 3. Our NLO results are drawn in solid curve and the $1\pi C_2$ in dashed curve. The PDS results obtained up to NLO are drawn both for the coefficients fit to the low-energy constants, a_0 and r_e (dotted line) [14] and for the results obtained by a global fit for the range $p \leq 200$ MeV (dot-dashed line). It might be worthy to note that the NLO in PDS and in our framework have different meanings, as the counting rules adopted are different. Nevertheless both theories have the same number of free parameters, so the comparison can be meaningful. From the figure, one can confirm that the cutoff theory is doing better than the PDS results in accuracy. A similar observation was made in [14]. Recently, the NNLO calculation in the PDS method (which is Q^2 order compared to the LO) was reported, both in a toy model [27] and in a real situation [29]. In [29], they have a good agreement up to $p = 400$ MeV by fitting the parameters over $p = 7 - 200$ MeV. Due to their fitting procedure, it is however meaningless to directly

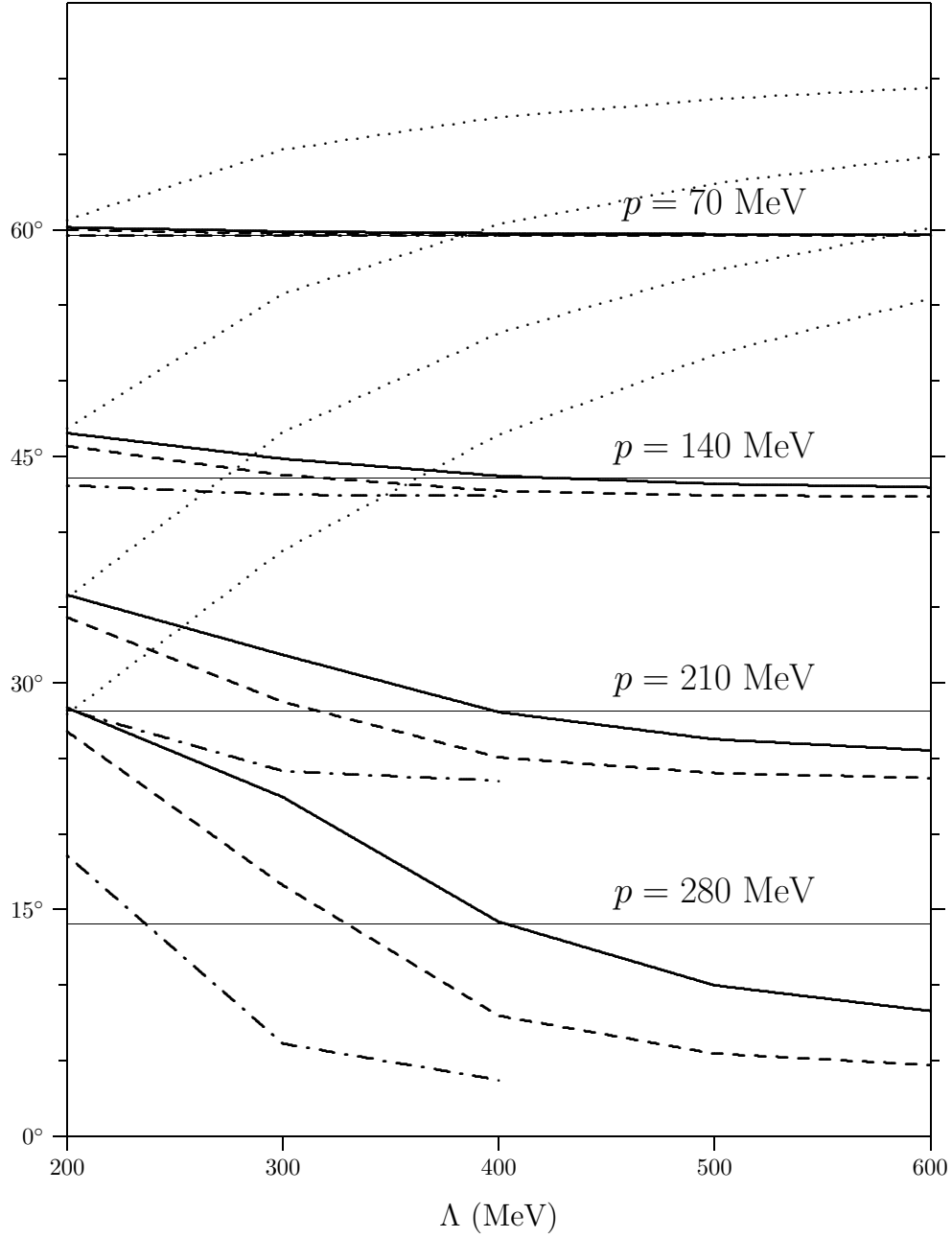


Figure 2: Phase shift with respect to Λ at $p = 70, 140, 210$ and 280 MeV. For each momentum, the LO, $1\pi C_2$, NLO and $2\pi C_4$ results are represented by dotted, dashed, solid and dotted-dashed curves.

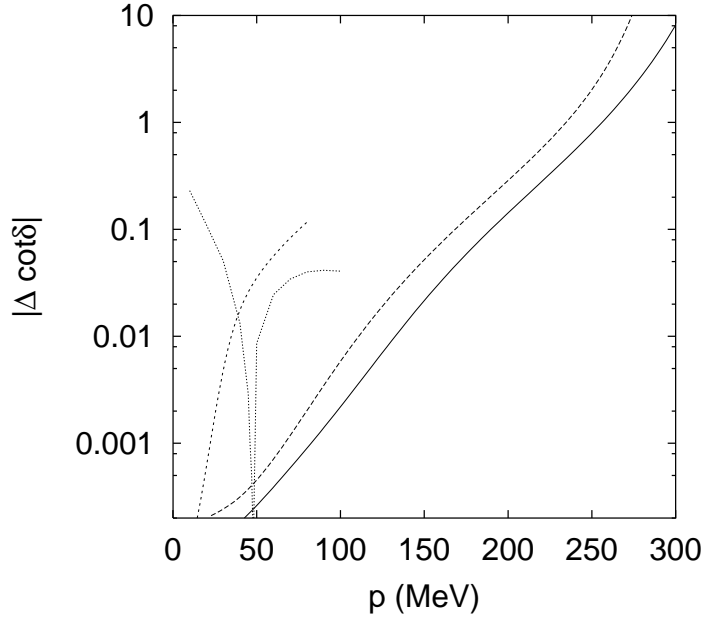


Figure 3: $|\Delta \cot \delta| (\equiv |\cot \delta_{\text{EFT}} - \cot \delta_{\text{NPWA}}|)$ with respect to the momentum p . Solid line is for NLO($= 2\pi C_2$) and dashed line for $1\pi C_2$. Dotted line is the results of PDS fitted to a_0 and r_e and dot-dashed line is the results for fit over $p \leq 200$ MeV.

compare their results to ours.^{#5} In applying their framework to the spin-one channels, they conclude that the nonperturbative treatment of potential is needed.

To conclude, we have demonstrated in this letter that the Λ scheme can be made markedly more successful for the scattering problem by going to the NLO order *a la* Weinberg. We have also shown the importance of the explicit presence of the pion exchange potential in NLO, extending the domain of validity as well as increasing the precision. The method is found to be as successful in scattering as in electroweak responses. It has also been discussed that there are missing parts in the TPEP, the subleading order contributions, which are expected to play a significant role in increasing the accuracy at high momentum region. While we are revising this paper, quite a similar work has been reported by Epelbaum, Glöckle and Meißner [21]. They have constructed the potential including up to the subleading TPEP (and the leading TPEP but with also Delta isobar). They have performed the calculation in momentum space, and applied to various channels as well as the $1S_0$ np scattering. And they have got similar conclusions to ours, including the importance of the subleading TPEP.

We are particularly grateful to Mannque Rho for his continuous supports and invaluable discussions during this work. Work of DPM and CHH is partially supported by the KOSEF through CTP of SNU and by the Korea Ministry of Education under contract No.

^{#5} As a consequences of the fitting, their low-energy constants are different from experiment. For example, the effective volume in their approach is $v_2 = -1.2 \text{ fm}^3$, which should be compared to the data -0.48 fm^3 .

99-2418 and BK-21.

References

- [1] T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Phys. Rev. **C58** (1998) 637; Nucl. Phys. **A646** (1999) 83; T.-S. Park, hep-ph/9803417.
- [2] T.-S. Park, K. Kubodera, D.-P. Min, M. Rho, Astrophys. Jour. **507** (1998) 443.
- [3] T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, nucl-th/9904053; nucl-th/9906005.
- [4] S. Weinberg, Phys. Lett. **B295** (1992) 114.
- [5] S. R. Beane, C. Y. Lee and U. van Kolck, Phys. Rev. **C52** (1995) 2914.
- [6] S.R. Beane, V. Bernard, T.-S. H. Lee, U.-G. Meißner and U. van Kolck, Nucl. Phys. **A618** (1997) 381.
- [7] S.R. Beane, M. Malheiro, D.R. Phillips and U. van Kolck, nucl-th/9905023.
- [8] U. van Kolck, Nucl. Phys. **A645** (1999) 273.
- [9] S. Weinberg, Phys. Lett. **B251** (1990) 288; Nucl. Phys. **B363** (1991) 3; Phys. Lett. **B295** (1992) 114.
- [10] INT-Caltech 1998 workshop proceedings, *Nuclear Physics with Effective Field Theory*, ed. R. Seki, U. van Kolck, and M.J. Savage, World Scientific, 1998
- [11] D.B. Kaplan, M.J. Savage and M.B. Wise, Phys. Lett. **B424**, 390 (1998).
- [12] M. Lutz, nucl-th/9906028.
- [13] C. Ordóñez and U. van Kolck, Phys. Lett. **B291** (1992) 459; U. van Kolck, Texas Ph.D. Dissertation (1993); C. Ordóñez, L. Ray and U. van Kolck, Phys. Rev. Lett. **72** (1994) 1982; Phys. Rev. **C53** (1996) 2086.
- [14] T.D. Cohen and J.M. Hansen, Phys. Rev. **C59** (1999) 13; D.R. Phillips and T.D. Cohen, nucl-th/9906091.
- [15] J. V. Steele and R. J. Furnstahl, Nucl. Phys. **A645** (1999) 439.
- [16] T.-S. Park, D.-P. Min and M. Rho, Phys. Rev. Lett. **74** (1995) 4153; Nucl. Phys. **A596** (1996) 515.
- [17] M. Robilotta and da Rocha, Nucl. Phys. **A615** (1996) 2086.
- [18] N. Kaiser, R. Brockmann and W. Weise, Nucl. Phys. **A625** (1997) 758.

- [19] M. C. M. Rentmeester, R. G. E. Timmermans, J. L. Friar and J. J. de Swart, Phys. Rev. Lett. **82** (1999) 4992.
- [20] J.L. Friar, nucl-th/9901082; U. van Kolck, nucl-th/9902015.
- [21] E. Epelbaum, W. Glöckle and Ulf-G. Meißner, nucl-th/9910064.
- [22] G.P. Lepage, nucl-th/9706029; talk given at the Workshop on “Nuclear Physics with Effective Field Theories,” INT, University of Washington, February 25-26, 1999.
- [23] V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester and J.J. de Swart, Phys. Rev. **C48** (1993) 792.
- [24] R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. **C51** (1995) 38.
- [25] T.D. Cohen and J.M. Hansen, Phys. Rev. **C59** (1999) 13.
- [26] S.R. Beane, T.D. Cohen and D.R. Phillips, Nucl. Phys. **A632** (1998) 445; K.A. Scadferri, D.R. Phillips, C.-W. Kao and T.D. Cohen, Phys. Rev. **C56** (1997) 679; Daniel R. Phillips and Thomas D. Cohen, Phys. Lett. **B390** (1997) 7.
- [27] G. Rupak and N. Shores, nucl-th/9902077.
- [28] J.-W. Chen, G. Rupak, M.J. Savage, nucl-th/9905002.
- [29] S. Fleming, T. Mehen and I.W. Stewart, nucl-th/9911001.